disabling the SST model in these flow regions. However, we also suggest that it is possible to correct this shortcoming by slightly modifying the heuristic used to "turn on" the SST feature of the model. Extensive recalibration of the model by validating against a large body of cases would be required before such a modification should be trusted for general applications.

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Impact of Compressibility on Mixing Downstream of Lobed Mixers

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Introduction

LOBED mixers are used in a number of applications to augment the mixing rate between co-flowing fluid streams. These devices enhance mixing by two mechanisms: the increased initial interface area associated with their convoluted trailing-edge shape and the introduction of streamwise vorticity.[1–5] The focus of this Note is an assessment of the impact of compressibility on the mixing associated with streamwise vorticity.

In this work, the compressibility of the mixing layers is characterized by the convective Mach number M_c and the high-speed (primary) stream inflow Mach number M_P. The convective Mach number is defined as $M_c = (U_p - U_s)/\bar{a}$, where U_p is the high-speed (primary) stream velocity, U_s is the low-speed stream velocity, and $\bar{a}$ is the mean inflow speed of sound. In planar shear layers, a threefold drop in the growth rate of planar shear layers has been observed[6] as the convective Mach number increases from 0 to 1.

In lobed mixer flows with a supersonic high-speed stream, in addition to the mixer-generated vorticity, streamwise vorticity may be generated downstream of the mixer trailing edge via the interaction of trailing-edge shocks and density gradients across the mixing layer. Depending on the density ratio between the high-speed and low-speed streams, this baroclinic-torque-generated vorticity can either enhance or retard the mixing rate.

Description of Approach

The impact of compressibility on the streamwise-vorticity-enhanced mixing process was assessed in a series of wind-tunnel experiments and with the aid of a numerical discrete vortex model.

Experimental

The experiments were conducted in the Supersonic Shear Flow Facility at United Technologies Research Center.[7,8] Two converging/diverging nozzles provided supersonic inflow Mach numbers of 1.3 or 2.4. A converging nozzle was used for the subsonic stream. The subsonic stream Mach number was varied by adjusting the inflow total pressure and mixing duct backpressure. The mixing duct was 10 by 10 cm in cross section and 64 cm long, and the inflow streams were of equal area. The two stream Mach numbers were varied in such a way that the velocity ratio remained constant, but the convective Mach number changed.

In addition to the compressibility of the mixing layers, the streamwise circulation shed from the trailing edge of the lobed mixers was independently controlled by the use of four different geometries: one planar splitter plate and three lobed mixers. Lobed mixers with either a 15- or a 25-deg ramp angle were designed to shed streamwise circulation; the third mixer was a convoluted plate with the same trailing-edge shape as the two lobed mixers but was not ramped and, therefore, shed little streamwise circulation. The mixers were of an approximately square lobe design with a height-to-wavelength ratio of $h^* = 1.25$. (The lobe wavelength was $\lambda = 0.31$ cm.) The nondimensional streamwise circulation shed by the two forced mixers were estimated by the scaling law developed by Barber et al.[9]:

$$\Gamma^* = \Gamma/\bar{U}h = 2\tan\alpha \tag{1}$$

where Γ is the circulation, $\bar{U}$ is the average inflow velocity of the two streams, h is the mixer height, and α is the ramp angle. The estimated nondimensional circulations for the 15- and 25-deg mixers were 0.5 and 0.9, respectively. The circulation shed by the convoluted plate mixer was estimated to be 0.1 based on Navier–Stokes computations performed by O'Sullivan[4] for similar geometries.

The compressibility of the mixing layer as defined by the convective Mach number and primary Mach number was controlled by varying the inflow Mach numbers of the two streams. The convective Mach number ranged from 0.3 to 0.8 and the primary stream Mach number from 1.3 to 2.4.

The experimental diagnostics included schlieren photographs, Mie scattering imaging, and pitot/static pressure surveys. Mie scattering sites were provided by seeding the supersonic stream with methanol liquid upstream of the plenum chamber.[10] Laser illumination was provided by a frequency-doubled Nd:YAG laser formed into a sheet roughly 5 cm wide and 1 mm thick. The planar Mie images were obtained with a charge-coupled device camera with an array of 576 × 384 pixels. The schlieren photographs and Mie images were used to develop a qualitative understanding of the mixing

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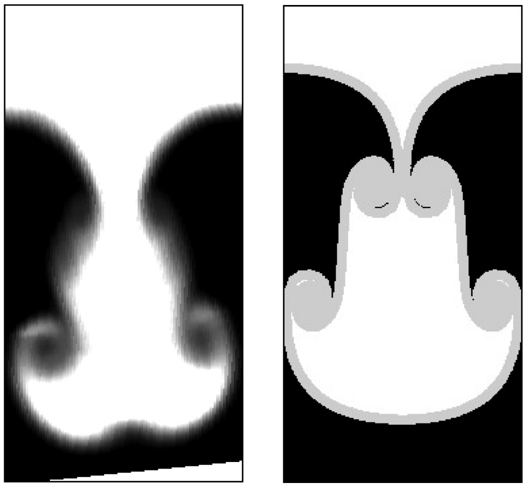


a) Mie scattering data b) Model scalar field

Fig. 1 Model scalar field and Mie scattering data crossflow interface structure comparison at $x^* = 2$ downstream of 15-deg forced mixer with $M_P = 2.4$ and $M_s = 1.6$.

process (see Ref. 8). The pitot/static surveys were used to measure the mixedness of the two streams quantitatively.

Compressible Vortex Model

To aid in the assessment of the compressibility effects, a discrete vortex model of the streamwise-vorticity-enhanced mixing process was developed. In the model, a number of viscous vortices are aligned along the mixer trailing edge to model the shed streamwise-vorticity distribution. The induced velocity fields of the vortices are then superposed to model the evolution of the interface with distance from the trailing edge. An empirical turbulence model, based on the planar shear layer scaling law, is utilized to estimate the diffusion rate of the streamwise and transverse vorticity, and a passive scalar is tracked to calculate the scalar mixedness. In Fig. 1, the experimental time-mean Mie image and the modeled scalar field are shown for the 15-deg forced mixer at a normalized downstream distance of $x^* = x/\lambda = 2$ with $M_P = 2.4$ and $M_s = 1.6$. The structure revealed by the Mie images can be seen to be qualitatively similar to that predicted by the vortex model, suggesting the utility of the model for the rapid assessment of the effectiveness of lobed mixer configurations in advance of a detailed computational fluid dynamics and/or experimental analysis.

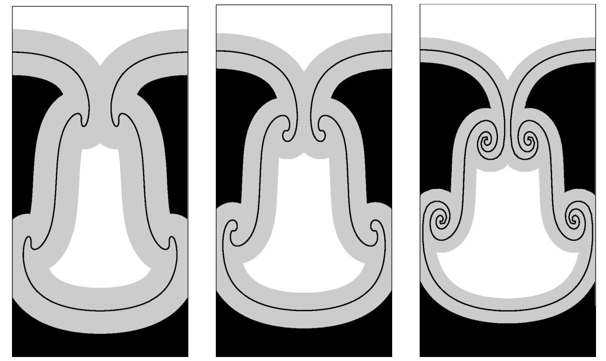
Two features were then added to this incompressible discrete vortex model to assess the impact of compressibility as characterized by the convective Mach number and primary stream Mach number[8,11]: 1) the use of the Papamoschou and Roshko scaling law for compressible shear layer mixing in the turbulent diffusion model and 2) the generation of vorticity downstream of the mixer trailing edge by the baroclinic torque exerted by the trailing-edge shock on the density gradient across the mixing layer.

Results

Impact of the Convective Mach Number

For the lobed mixer flows of the current experiments, a decreasing mixing rate with increasing M_c was observed in pitot/static pressure survey data. In this estimation, the mixing measure used was $\Psi(x) = [\bar{P}_t^m|_{\mathrm{TE}} - \bar{P}_t^m(x)]/[\bar{P}_t^m|_{\mathrm{TE}} - \bar{P}_t^m|_{\mathrm{mixed}}]$, where P is the mass-flow-averaged total pressure at the mixer trailing edge, at an indicated downstream station x, and at the fully mixed state at constant area. This normalized mixing measure varies between zero at the unmixed trailing edge and unity at the fully mixed conditions; the uncertainty in Ψ is estimated to be ± 0.02.

The normalized mass-averaged total pressure was determined experimentally for the flat plate, convoluted plate, and 15-deg forced mixer for two different convective Mach numbers, $M_c = 0.43$ and $M_c = 0.58$, at approximately the same low-speed-to-high-speed velocity ratio, $r = 0.25$ and $r = 0.20$, respectively. For all three configurations, the value of Ψ decreased somewhat with increasing convective Mach number. The measured values of the mixing parameter for the lower and higher convective Mach numbers were, respectively, as follows: For the flat plate, $\Psi = 0.35$ and 0.28; for the convoluted plate, $\Psi = 0.47$ and 0.42; for the 15-deg forced mixer, $\Psi = 0.64$ and 0.59.



a) $M_c = 0.06$ b) $M_c = 0.46$ c) $M_c = 0.96$

Fig. 2 Computed fully mixed scalar fields $x^* = 2$ downstream of 15-deg forced mixer at three convective Mach number, $r = 0.2$ and $s = 1.0$.

Thus, in mixing layers downstream of lobed mixers, a mixing penalty does appear to be associated with increased convective Mach number at constant velocity ratio. This observed decrease is in qualitative agreement with the planar shear layer results of Papamoschou and Roshko[6] (also see Ref. 3).

The mechanism for the decreasing mixing rates with increasing Mach number M_c lies partially with the decreased rate of growth of the shear layer on the interface between the coflowing streams. This decrease in shear layer growth is shown in Fig. 2 based on the vortex model.

The computed mixing augmentation associated with the streamwise vorticity also varies with the convective Mach number. This mixing augmentation at a particular axial location and convective Mach number can be defined by $\Delta\mathfrak{M}_{\mathrm{sc}} = (\mathfrak{M}_{\mathrm{sc}}|_{\Gamma^* \neq 0} - \mathfrak{M}_{\mathrm{sc}}|_{\Gamma^* = 0})/\mathfrak{M}_{\mathrm{sc}}|_{\Gamma^* = 0}$, where $\mathfrak{M}_{\mathrm{sc}}$ is the mixedness of a scalar quantity such as species concentration for either the case of zero or nonzero streamwise circulation, $\Gamma^* = 0$ or $\Gamma^* \neq 0$, respectively. The quantity $\Delta\mathfrak{M}_{\mathrm{sc}}$ varies between zero at the unmixed trailing edge and unity for the fully mixed condition.

The computed mixing augmentation (but not necessarily the overall mixedness) associated with streamwise vorticity at any location downstream of the mixer increases with increasing convective Mach number, as shown in Fig. 3. The physical reason for the increased importance of streamwise vorticity with increasing Mach number M_c lies with the decreasing shear layer growth rates and corresponding decrease in the rate of diffusion of the streamwise vorticity away from the interface. Thus, whereas a decreased overall mixing rate is associated with the decreased levels of turbulent diffusion that accompanies an increasing convective Mach number,[8] the mixing augmentation associated with streamwise vorticity increases with increasing Mach number M_c.

Impact of Primary Mach Number

In supersonic–subsonic mixer flows, shocks located in the vicinity of the mixer trailing edge can interact with the density gradient across the mixing layer to generate levels of streamwise circulation comparable to that shed by the mixer.

Over the range of primary Mach numbers investigated here, when the trailing-edge static pressures were matched, the primary flow was turned back into approximately the axial direction as verified by the angle of the bottom of the mixing layer in schlieren photographs. As the secondary/primary static pressure ratio was decreased, the turning of the primary trough flow at the trailing edge became less abrupt. On the other hand, as the static pressure ratio was increased, the trailing-edge shock strengthened, and in some cases the shock

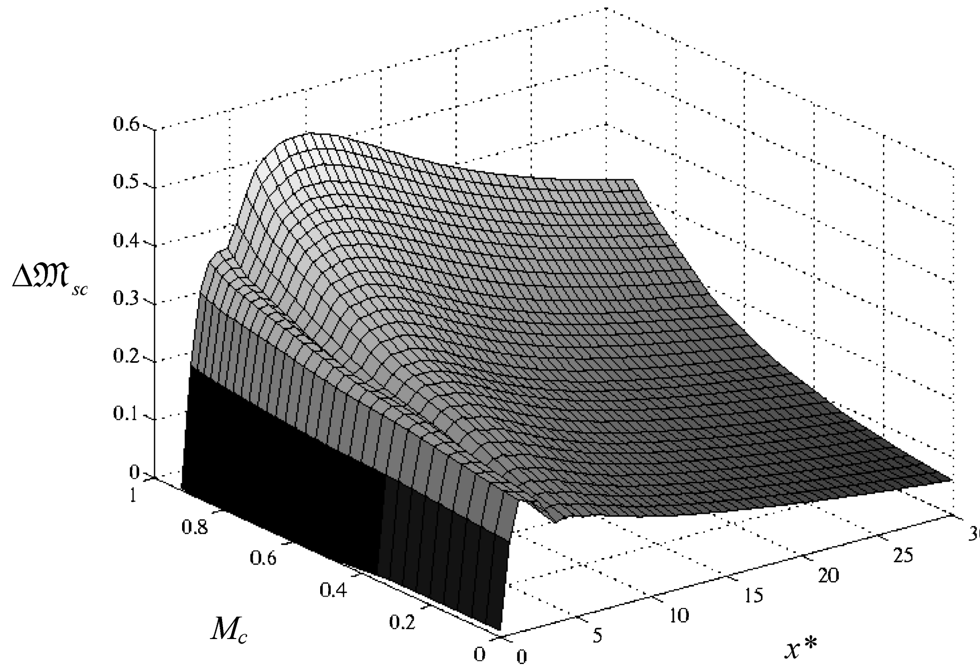


Fig. 3 Scalar mixing augmentation associated with the streamwise vorticity shed by 15-deg forced mixer, Γ* = 0.5, vs convective Mach number and distance from trailing edge for velocity ratio *r* = 0.2.

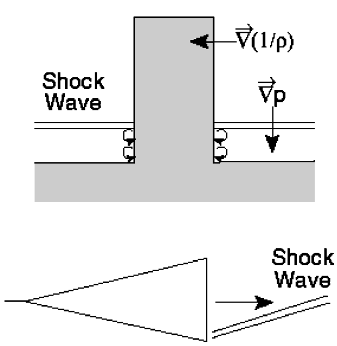


Fig. 4 Generation of streamwise vorticity by baroclinic torque associated with interaction of trough shock and density gradient across the mixing layer; low-speed-to-high-speed density ratio less than 1.

moved upstream into the trough causing the flow separation within the lobed mixer.

The interaction of the shock and the density gradient that can exist across the mixing layer can result in the generation of streamwise circulation. In mixing layers where the low-speed-to-high-speed stream density ratio is not unity ($s \neq 1$), the mechanism for the generation of the circulation is that shown in Fig. 4 (here for the case $s < 1$). The secondary stream sees approximately the same pressure gradient due to the shock wave turning as the primary stream, and this pressure gradient turns the secondary flow farther off-axis, increasing the streamwise circulation. An analytical model of this shock/mixing layer interaction was derived to estimate the streamwise circulation generated downstream of the trailing edge of the mixers.[8]

The rate of change of the streamwise circulation with distance from the trailing edge of the mixer is found from

$$\frac{\mathrm{D}\Gamma}{\mathrm{D}(x/\bar{U})} = -\iint \left(\frac{\nabla\rho \times \nabla p}{\rho^2}\right) \cdot \hat{n}\,\mathrm{d}A \tag{2}$$

where ρ and p are, respectively, the static density and pressure and $\hat{n}$ is the unit outward vector normal to the differential area $\mathrm{d}A$. If the trough flow is assumed to leave the mixer at the metal angle α and turn back into the axial direction through an angle θ due to the oblique shock, and if the static pressure and density are assumed to vary linearly across the shock and mixing layer, respectively, Eq. (1) may be integrated from the trailing edge of the mixer to the downstream end of the shock to estimate the total circulation generated by the shock. The ratio of the shock-induced circulation to that generated by the mixer is then given by

$$\Gamma_{\text{shock}}/\Gamma_{\text{mixer}} = \left(-8\big/\gamma M_p^2\right)ab$$

$$a = (s-1)/[(s+1)^2(r+1)^2](\pi_{\text{shock}} - 1)$$

$$b = 1/\tan\alpha(\tan\alpha + \tan\theta) \tag{3}$$

where π_{shock} is the static pressure ratio across an oblique shock with an inflow Mach number of M_P and a flow turning angle of θ due to the shock wave. The ratio of the calculated shock-generated to mixer-shed circulation is shown in Fig. 5 vs the density ratio for a lobe angle of 15 deg, a high-speed Mach number of 1.6, and velocity ratios of 0.2, 0.4, 0.6, 0.8, and 1.0. From Fig. 5 and Eq. (3), it may be seen that if the density ratio s is less than unity, circulation is generated in the same sense as that generated by the mixer. On the other hand, if $s > 1$, circulation is generated in the opposite sense as that generated by the mixer.

To assess the impact of this circulation generation on the mixing rate downstream of these mixers, the baroclinic torque model was implemented in the discrete vortex simulations described earlier. The location of the shock was determined from a simplified model of the trough flow. Given the two stream Mach numbers and total temperature ratio, the circulations of the individual vortices were adjusted by Eq. (3) as they passed through the shock.

Based on the scalar fields, a decrease in the density ratio results in an increasing effectiveness of the mixer as additional circulation is generated downstream of the trailing edge. This additional circulation increases the rate of stretching of the interface. On the other hand, an increase in the turning angle θ and the corresponding increase in the density ratio s results in the generation of circulation in the sense opposite to that shed by the mixer. This shock-generated circulation counteracts that generated by the mixer, and a decrease in the rate at which the interface is stretched results. In any case, the model predicts that the forced mixer mixing rate will always be equal to (if all of the vorticity is canceled) or higher than that downstream of the convoluted plate.

Note that the generation of the streamwise circulation downstream of the mixer is driven by the trailing-edge shock-induced pressure gradients and the density gradient across the mixing layer. This circulation will impact the mixing rate via its influence on

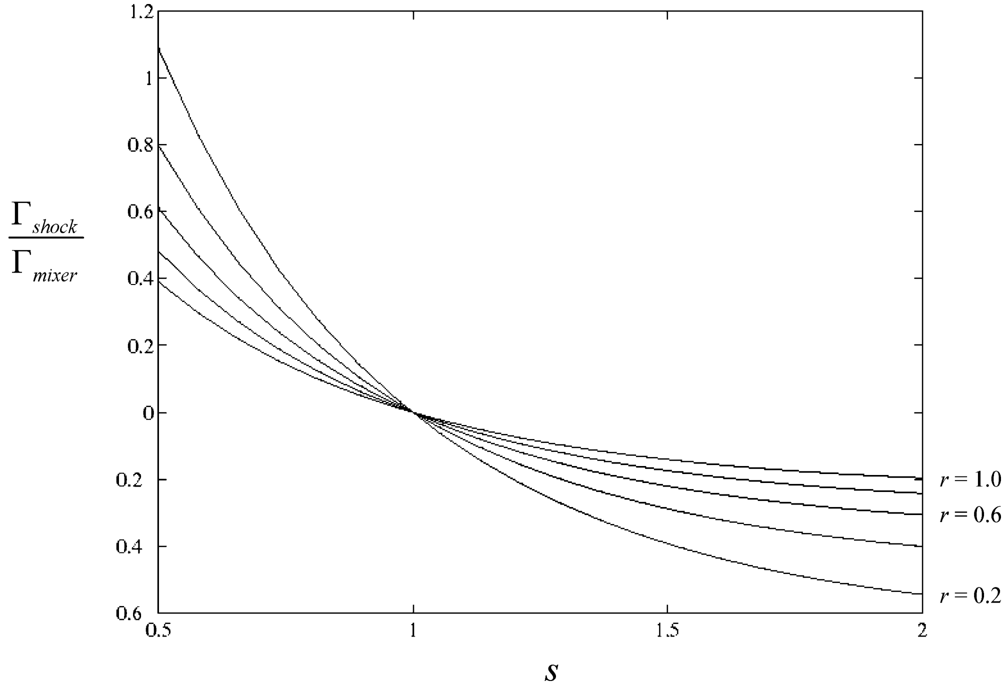


Fig. 5 Shock/mixer circulation ratio vs low-speed-to-high-speed density ratio s, for 15-deg forced mixer with $M_P = 1.6$.

the rate at which the crossflow interface is stretched. This mixing mechanism is fundamentally different from that associated with the convective Mach number, turbulent diffusion across the interface, and hence, the convective Mach number does not appear explicitly in the preceding analysis.

Summary

In a series of experiments and modeling efforts, a twofold impact of compressibility on the streamwise-vorticity-enhanced mixing process downstream of lobed mixers has been demonstrated:

1) Reduced shear growth rates associated with high convective Mach number give the streamwise vorticity more time to stretch the interface between the two streams, and hence, streamwise vorticity is more effective in compressible regimes than in incompressible ones.

2) Vorticity may be generated downstream of the mixer by the interaction of trailing-edge shocks and density gradients across the mixing layer. Depending on the density ratio between the streams, this vorticity generation may either enhance or reduce the mixing rate.

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